IDENTIFICATION OF LEARNING OBSTACLES AND PREPARATION OF DIDACTIC DESIGN OF TECHNOLOGY-ENHANCED LEARNING INTEGRATED HYPOTHESIS IN PYTHAGOREAN THEOREM MATERIAL

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Abstract

Learning the Pythagorean Theorem continues to present various learning challenges that may limit students' conceptual understanding and their ability to apply the theorem across different problem-solving situations. Therefore, this study sought to explore the learning obstacles encountered by students when studying the Pythagorean Theorem and to formulate a hypothetical didactical design incorporating the Technology-Enhanced Learning (TEL) approach. The study employed the Didactical Design Research (DDR) method, focusing on the prospective analysis stage. The research participants consisted of a mathematics teacher and Grade VIII J students of SMP Negeri 03 Brebes as the pre-implementation class. Data were collected through lesson plan analysis, teacher interviews, diagnostic tests, and student interviews. The findings revealed three types of learning obstacles: ontogenic obstacles related to students' learning readiness and their ability to interpret problem information; epistemological obstacles associated with limited conceptual understanding of the Pythagorean Theorem across different representations and contexts; and didactical obstacles influenced by limited instructional time, which reduced students' opportunities to explore concepts in depth. Based on these findings, a hypothetical didactical design was developed by integrating the TEL approach through interactive PowerPoint presentations, GeoGebra, learning videos, QR codes, and Liveworksheet, accompanied by predictions of students' responses and didactical-pedagogical anticipations. The resulting hypothetical didactical design is expected to serve as an alternative instructional design that accommodates students' learning obstacles in learning the Pythagorean Theorem.

Keywords: Didactical Design Research; learning obstacles; Pythagorean Theorem; Technology-Enhanced Learning; hypothetical didactical design;

1 INTRODUCTION

Mathematics is an essential subject that contributes to the development of students' logical reasoning, critical thinking, systematic thinking, and problem-solving abilities. Among the topics taught at the junior high school (SMP) level, the Pythagorean Theorem serves as an important foundation for understanding various concepts in geometry. This material not only requires students to understand the relationship between the sides of a right triangle, but also to be able to apply the concept to various forms of representation and contextual problems. However, in practice, the Pythagorean Theorem material is still one of the materials that is considered difficult by students because it involves the ability to visually representation, reasoning, and relating various geometric concepts.

Various studies show that students still have difficulty in understanding and solving problems in the material of the Pythagorean Theorem. Research Rahmawati et al., (2022) shows that students still have difficulties in solving mathematical problems in the Pythagorean Theorem material, especially in understanding problems and determining the right solution strategy. Furthermore, the research Permatahati et al., (2022) analyze students' mistakes in solving Pythagorean Theorem problems and show the existence of various mistakes made by students in the problem-solving process. The findings show that students' difficulties in the Pythagorean The Pythagorean Theorem involves more than the ability to apply formulas. Students also need to understand the underlying concepts and use appropriate problem-solving strategies. Meanwhile, research Ikhtiara et al., (2026) regarding didactic design in the material, the Pythagorean Theorem shows the importance of designing learning by taking into account the understanding of students' concepts and obstacles that may arise during the learning process. The three studies show that a learning design is needed that pays attention to students' learning difficulties and obstacles so that the Pythagorean Theorem learning process can take place more meaningfully and in accordance with students' needs.

However, the results of the study show that previous research generally only focused on identifying student errors, analyzing learning obstacles, or applying technology-based learning media separately. There are still few studies that integrate the results of the identification of learning obstacles as the basis for the preparation of hypothesis didactic designs combined with the Technology-Enhanced Learning (TEL) approach in the Pythagorean Theorem material. In other words, previous research has not developed much learning design that is compiled based on the characteristics of students' learning barriers while utilizing technology as part of the didactic situation. This condition shows that there is a research gap that is the basis for this research.

A preliminary study conducted at SMP Negeri 03 Brebes through the analysis of learning tools, interviews with mathematics teachers, and diagnostic tests showed that students still had difficulties in understanding the concept of the Pythagorean Theorem in various contexts. Some students have not been able to determine the relationship between the sides of a right triangle correctly, have difficulty applying the inverse of the Pythagorean Theorem, and have not been able to solve the story problem or determine the comparison of the sides of a right triangle with a special angle. In addition, the results of interviews with teachers show that the limited allocation of learning time causes students to explore concepts in depth and obtain a variety of learning experiences are still limited. The findings indicate that there are learning obstacles that need to be identified before learning preparation is carried out.

One approach that can be used to overcome this problem is Didactical Design Research (DDR). Through DDR, the learning process begins with an analysis of didactic situations to identify learning obstacles experienced by students as a basis for designing didactic situations. According to Suryadi (2013), the didactic design that was prepared then considered the prediction of student responses as well as pedagogical didactic anticipation to accommodate various possible responses that arise during the learning process. The study of learning obstacles also places learning barriers as one of the important aspects that need to be considered in designing learning to fit the process of constructing students' knowledge (Brousseau, 2002).

To make the learning process more engaging and support students' understanding of geometric concepts, this study integrates the didactic design with a Technology-Enhanced Learning (TEL) approach. The design uses digital learning tools, including interactive PowerPoint presentations, GeoGebra, instructional videos, QR Codes, and Liveworksheets. These tools provide students with opportunities to explore learning materials through different digital representations and interactive activities. Such technology integration can support meaningful learning experiences and facilitate students' understanding of mathematical concepts (Beetham & Sharpe, 2013; Karlinger & Erbaş, 2024). The design also considers the specific learning obstacles identified in the study. Thus, the learning activities are structured to support students in developing their understanding step by step.

Based on these considerations, this study has two objectives. First, it identifies students' learning obstacles in learning the Pythagorean Theorem. Second, it develops a hypothetical didactical design that integrates the Technology-Enhanced Learning (TEL) approach based on the identified learning obstacles. The novelty of this study lies in the development of a hypothetical didactical design that directly responds to students' identified learning obstacles. The design also incorporates digital learning tools to address specific conceptual difficulties in learning the Pythagorean Theorem.

2 METHODOLOGY

This research paper uses a qualitative approach with the Didactical Design Research (DDR) method developed by Suryadi. In this article, the research is focused on the prospective analysis stage, namely identifying students' learning obstacles as the basis for the preparation of hypothetical didactic designs that are integrated with the Technology-Enhanced Learning (TEL) approach. The research stage begins with an analysis of learning tools (teaching modules/lesson plans) to find out the learning outcomes, flow of learning objectives, materials, learning models, media, and assessments that have been used by teachers in the Pythagorean Theorem material. Furthermore, interviews were conducted with mathematics teachers to obtain information about the learning process and difficulties experienced by students. Based on the results of the analysis, the researcher compiled a diagnostic test which was then validated by two mathematics education lecturers and two mathematics teachers. Diagnostic tests were given to students in grade VIII J as a pre-implementation class, then the results were analyzed to identify learning obstacles. The analysis was deepened through interviews with several students who were selected based on the variation of answers. The results of the identification of learning obstacles were then used as the basis for compiling

a hypothesis didactic design that contained learning objectives, didactic situations, predictions of student responses, pedagogical didactic anticipation, and integration of the TEL approach.

The research was carried out at SMP Negeri 03 Brebes in the even semester of the 2025/2026 Academic Year. The research subjects consisted of one mathematics teacher and 25 students of grade VIII J as a pre-implementation class. The class selection is carried out purposively with the consideration that students have studied the material of the Pythagorean Theorem so that they can provide information about the form of learning obstacles experienced. The material studied included the Pythagorean Theorem, the inverse of the Pythagorean Theorem, the application of the Pythagorean Theorem to flat and spatial construction, the solution of story problems, and the comparison of sides on a right triangle with a special angle.

The research instruments consist of learning tool analysis, teacher interview guidelines, diagnostic tests, student interview guidelines, and hypothesis didactic design preparation sheets. Learning tool analysis is used to identify the suitability between learning outcomes, learning goal flows, materials, learning models, media, assessments, and learning activities. Teacher interviews are used to obtain information about the implementation of learning and students' learning difficulties in the Pythagorean Theorem material. The diagnostic test is in the form of description questions that are prepared based on learning indicators to identify learning obstacles. The student interview guidelines help the researcher examine students' reasoning when completing the diagnostic test. The didactic design sheets guide the researcher in developing learning activities based on the identified learning obstacles. All instruments were validated through expert judgment by two mathematics education lecturers and one mathematics teacher by paying attention to the suitability of indicators, materials, language, and research objectives.

The diagnostic test indicators are compiled based on the Learning Objective Flow (ATP) of the Pythagorean Theorem material as presented in Table 1.

Table 1. Diagnostic Test Indicatiors.

No	Indicator
1	Stating the Pythagorean Theorem.
2	Calculate the length of the sides of a right triangle using the Pythagorean Theorem.
3	Presents the opposite of the Pythagorean Theorem.
4	Applying the Pythagorean Theorem to a flat building.
5	Applying the Pythagorean Theorem to the construction of space.
6	Applying the Pythagorean Theorem to story problems.
7	Determine the oblique side of a right triangle with a special angle.

The study analyzed the data using qualitative techniques. The analysis followed three stages: data reduction, data presentation, and conclusion drawing. During data reduction, the researcher selected relevant findings from the learning instrument analysis, diagnostic tests, and teacher and student interviews. The researcher then presented the data through narrative descriptions, indicator-based tables, and selected samples of students' answers. This presentation helped the researcher examine patterns in students' responses and identify specific learning difficulties.

The researcher drew conclusions by examining the identified learning obstacles. The analysis focused on ontogenic, epistemological, and didactical obstacles. These findings provided the basis for developing the hypothetical didactical design. The researcher also applied data triangulation to strengthen the credibility of the findings. The triangulation process involved comparing evidence from learning instrument analysis, diagnostic tests, teacher interviews, and student interviews.

3 RESULTS AND CONCLUSIONS

3.1 Results

3.1.1 Identification of Learning Obstacles in the Pythagorean Theorem Material

The initial stage of research was carried out through the analysis of learning tools, interviews with mathematics teachers, the administration of diagnostic tests to students of grade VIII J, and interviews with several students. The four activities were carried out sequentially to obtain an overview of the learning obstacles experienced by students in the Pythagorean Theorem material. The results of the identification are then used as a basis for the preparation of a hypothesis didactic design that is integrated with the Technology-Enhanced Learning (TEL) approach.

The results of the diagnostic test showed that the level of student mastery of each indicator of the Pythagorean Theorem material still varied. The percentage of achievement of each indicator is presented which has been compiled by the researcher as follows.

Table 2. Diagnostic Test Indicators.

No.	Indicator	Percentage of Students Reaching KKTP (%)
1	Stating the Pythagorean Theorem	76%
2	Calculating the Length of the sides of a right triangle	88%
3	Presenting the inverse of the Pythagorean Theorem	72%
4	Applying the use of the Pythagorean Theorem to flat buildings	88%
5	Applying the Use of the Pythagorean Theorem to building space	96%
6	Applying the Pythagorean Theorem to the story problem	32%
7	Determine the oblique side of a right triangle with a special angle.	24%

Based on Table 2, the indicator applying the Pythagorean Theorem to the building of space obtained the highest achievement percentage of 96%, thus showing that the indicator has reached the KKTP. The indicator calculating the length of the sides of a right triangle and applying the Pythagorean Theorem to a flat build also shows a high percentage of achievement, each of 88%, so that both indicators have also achieved the KKTP. In addition, the indicator states that the Pythagorean Theorem obtained an achievement percentage of 76%, which indicates that the indicator has reached the KKTP. Meanwhile, the indicator presents the opposite of the Pythagorean Theorem with a percentage of 72%, so it has not yet reached the KKTP. However, some students have been able to determine the type of triangle based on the concept of the inverse of the Pythagorean Theorem, but there are still errors in understanding the relationship between the lengths of sides so that the solution obtained is not completely correct.

In contrast to these indicators, the indicator applies the Pythagorean Theorem to the story problem and determines the oblique side of the right triangle with special angles, obtaining a percentage of achievement of 32% and 24%, respectively, so that it has not reached the KKTP. These results show that most students still experience difficulties when the concept of the Pythagorean Theorem is applied to contextual problems and to material that requires mastery of the concept of right-angled triangle comparison with special angles.

The results of the diagnostic test were then analyzed in more depth through the analysis of student answers and interviews to identify the forms of learning obstacles experienced by students as a basis for the preparation of hypothesis didactic designs.

3.1.2 Ontogenic Barriers

Ontogenic obstacles are found in students who are not able to understand the information in the problem in its entirety so that they have difficulty determining the first step of completion. This condition is seen when students are still confused about identifying known and questioned information even though the basic concepts of the Pythagorean Theorem have been studied beforehand.

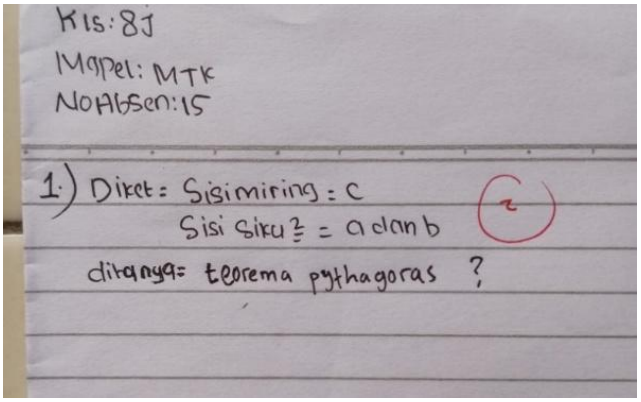

Figure 1. Example of a student's answer that shows an ontogenic barrier.

Based on Figure 1, it can be seen that students have not been able to connect the information contained in the question with the concept of the Pythagorean Theorem correctly. The results of the interviews showed that students were still hesitant to determine the completion step because they did not understand the relationship between elements in the right triangle. This condition shows that students' learning readiness has not developed optimally so that they still need learning assistance (scaffolding) in the form of triggering questions, gradual directions, and conceptual visualization in order to be able to build understanding independently. These findings are in line with Vygotsky's Zone of Proximal Development theory which states that students need appropriate assistance to reach their potential abilities.

3.1.3 Epistemological Barriers

Epistemological barriers are seen when students have understood the calculation procedure, but have not been able to apply the concepts to different contexts. One form of such resistance appears in the inverse indicator of the Pythagorean Theorem.

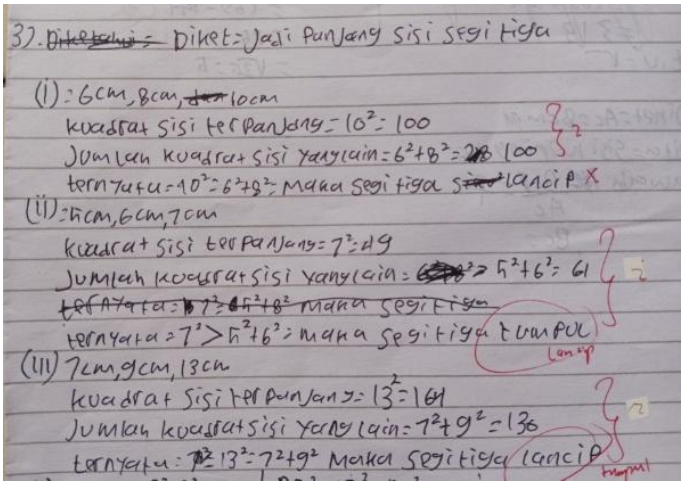

Figure 2. Example of a student's answer that shows the epistemological obstacle to the concept of the inverse of the Pythagorean Theorem.

In Figure 2, it can be seen that students have carried out the process of calculating the squares of the sides of the triangle, but have not been able to connect the results of the calculation with the classification of the types of triangles. This shows that students' understanding is still oriented towards calculation procedures, while the conceptual meaning of the inverse Pythagorean Theorem is not yet fully understood. These findings indicate that the concepts that students have are still limited to certain contexts so that when faced with different situations, students have difficulty in determining the right solution.

Epistemological obstacles are also found in the indicators of the application of the Pythagorean Theorem in the story problem.

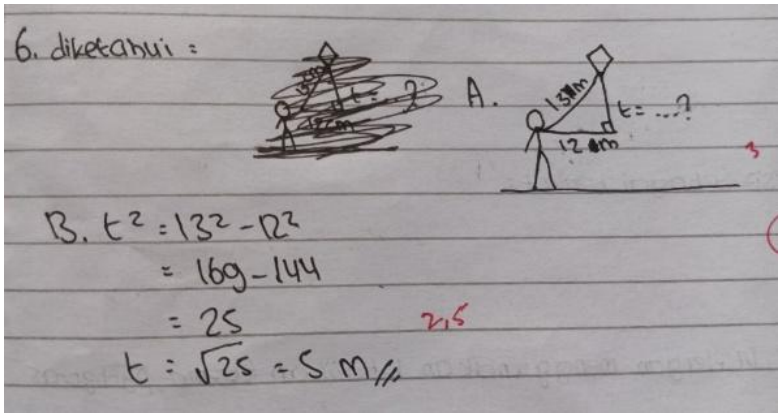

Figure 3. Example of student answers to indicators of the application of Pythagorean Theorem in story problems.

Based on Figure 3, Several students still struggled to relate the information provided in the problem to the appropriate application of the Pythagorean Theorem. Even though the necessary information is available, students still have difficulty determining the relationship between the data so that the completion is not appropriate. This condition shows that students are not able to transfer the concepts that have been learned into contextual problems.

Similar obstacles are also found in indicators determining the oblique side of a right-hand triangle with a special angle.

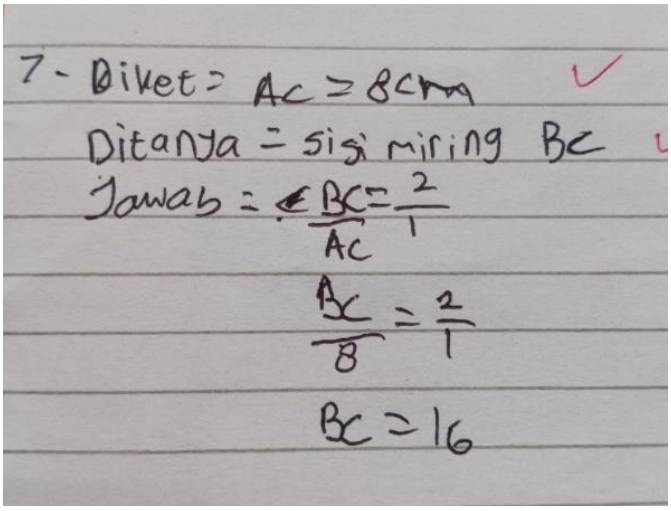

Figure 4. Example of student answers on a right-angled triangle indicator with a special angle.

Most students struggled to determine the side ratios in special right triangles with angles of 30°, 60°, and 90°, as well as 45°, 45°, and 90°. Interview findings indicated that students often applied the Pythagorean Theorem directly without first identifying the relevant side ratios. This finding suggests that their understanding remains focused on formula application. As a result, students encountered difficulties when solving problems that required them to combine the Pythagorean Theorem with the properties of special right triangles.

3.1.4 Didactic Barriers

Obtained through the analysis of learning tools and interviews with mathematics teachers. The results of the interviews show that the delivery of learning materials still focuses more on explaining concepts and sample questions given by teachers, while providing a variety of problems that can help students explore and deepen their understanding of concepts is still not done optimally. This condition causes students' learning experience to tend to be limited to following the explanations and examples given, so that students have not had enough opportunities to face various forms of problems that can train the ability to identify information, connect concepts, reason, and apply concepts in different situations.

The three forms of learning obstacles show that students' learning difficulties are not only influenced by learning readiness and mastery of concepts, but also by the learning experience gained during the learning process. Therefore, a learning design is needed that is able to anticipate various possible student responses through the presentation of various didactic situations and in accordance with the characteristics of the learning barriers that have been identified. The learning design is expected to provide opportunities for students to explore concepts through various situations and problems, so that the understanding formed does not only depend on the examples and explanations provided by the teacher.

3.2 Discussion

3.2.1 Preparation of Hypothetical Didactic Design

The identified learning obstacles provide a basis for developing the hypothetical didactical design for teaching the Pythagorean Theorem. The didactic design is prepared based on the stages of Didactical Design Research (DDR) by integrating the Technology-Enhanced Learning (TEL) approach. Each learning activity is designed by considering the possibility of student responses and pedagogical didactic anticipation that teachers can do if learning obstacles arise during learning.

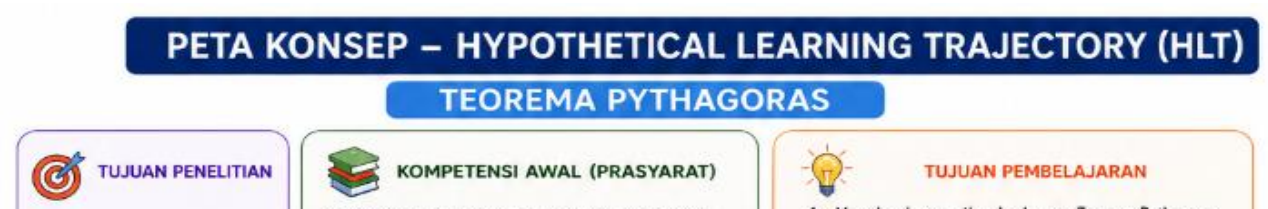

Figure 5. Hypothetical Learning Trajectory.

The hypothesis didactic design was arranged in three meetings according to the Learning Objectives Flow. The first meeting focused on understanding the concept of Pythagorean Theorem and its inverse through the use of interactive PowerPoint and GeoGebra to help visualize the relationships between the sides of a right triangle. The second meeting was designed to train students' ability to apply Pythagorean Theorem to flat and build space through learning videos, group discussions, and LKPD equipped with QR Codes. Furthermore, the third meeting focused on the material of right triangles with special angles through the presentation of various contextual examples and gradual exercises.

The integration of the TEL approach in didactic design not only serves as a medium for information delivery, but also as a scaffolding tool to help students build a gradual understanding of concepts in accordance with the principles of the Zone of Proximal Development (ZPD). Through the use of interactive media, concept visualization, and collaborative activities, didactic design is expected to be able to accommodate the various characteristics of learning obstacles that have been identified so that learning becomes more meaningful and in accordance with students' learning needs.

4 COUNCLUSION

This study identifies three types of *learning obstacles* in the Pythagorean Theorem material, namely ontogenic, epistemological, and didactic obstacles. Ontogenic barriers are related to the readiness of students to understand information and determine completion steps. Epistemological obstacles are shown by the limitations of students in applying the concept of the Pythagorean Theorem to various contexts, especially in the inverse of the Pythagorean Theorem, the story problem, and the right triangle with a special angle. Meanwhile, didactic barriers arise because learning still focuses on explaining the material and sample questions given by the teacher, so that students have not gained diverse learning experiences to explore concepts and apply them to various problem situations.

Based on the identified learning obstacles, the researcher developed a hypothetical didactical design that incorporates the Technology-Enhanced Learning (TEL) approach. The design uses interactive PowerPoint presentations, GeoGebra, instructional videos, QR Codes, and Liveworksheets to address specific learning needs in the Pythagorean Theorem. This didactic design was prepared by considering the characteristics of *learning obstacles*, predictions of student responses, and pedagogical didactic anticipation so that it is expected to be an alternative reference for teachers in designing Pythagorean Theorem learning that is more in line with students' learning needs.

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REFERENCES

Ahmad, H. (2023). Analisis kesulitan dalam menyelesaikan soal-soal Teorema Phytagoras pada peserta

didik. *ELIPS: Jurnal Pendidikan Matematika, 4*(1), 15–27.

Agustin, R. D., Susiswo, Sa'dijah, C., & Sukoriyanto. (2024). Obstacles student mathematics semantic representation to solving geometry problems. *AIP Conference Proceedings*. https://doi.org/10.1063/5.0194841

Bariyah, N., Prabawanto, S., & Dahlan, J. A. (2025). Learning obstacles and students' difficulties in solving the problem of Pythagorean theorem: A systematic literature review. *Jurnal Pendidikan MIPA, 25*(4), 1939–1960.

Beetham, H., & Sharpe, R. (Eds.). (2013). *Rethinking pedagogy for a digital age: Designing for 21st century learning* (2nd ed.). Routledge.

Brousseau, G. (2002). *Theory of didactical situations in mathematics: Didactique des mathématiques, 1970–1990*. Kluwer Academic Publishers.

Dahlan, J. A., Prabawanto, S., Bariyah, N., & Nti, S. J. (2025). The mechanism of didactical obstacles in the Pythagorean theorem: From visual rigidity to procedural failure. *Jurnal Pendidikan MIPA, 26*(4), 2495–2517. https://doi.org/10.23960/jpmipa.v26i4.pp2495-2517

Ikhtiara, A., Rachman, H., Rosyidi, A. H., & Sholikin, N. W. (2026). *Analisis Kesulitan Belajar Siswa Kelas VIII dalam Menyelesaikan Soal Cerita Teorema Pythagoras*. *2*, 1–16. https://research.iecjournal.com/index.php/IEC-JEI/article/view/8?utm_source

Permatahati, I., Rahmawati, N. D., & Aini, A. N. (2022). *Analisis Kesalahan Siswa dalam Menyelesaikan Soal Matematika Materi Teorema Pythagoras Ditinjau dari Kemampuan Berpikir Kritis*. *4*(5), 401–406. https://journal.upgris.ac.id/index.php/imajiner/article/view/10996?utm_source

Rahmawati, A., Warmi, A., & Marlina, R. (2022). *Analisis Kemampuan Pemecahan Masalah Matematis Siswa SMP Pada Materi Teorema Pythagoras*. *06*(01), 365–374. https://j-cup.org/index.php/cendekia/article/view/1012?utm_source

Rahmi, L., Yulianti, K., & Prabawanto, S. (2022). Students' learning obstacles on the topic of Pythagorean theorem. *AIP Conference Proceedings, 2468*, 070036. https://doi.org/10.1063/5.0102556

Ramdani, R. R., & Hidayati, K. (2026). Technology-enhanced learning (TEL): How does it affect pre-service mathematics teachers? *Jurnal Eksakta Pendidikan, 10*(1). https://doi.org/10.24036/jep/vol10-iss1/1046

Sandra, Y., & Suryadi, D. (2025). Students' learning obstacles and didactical anticipation in learning the Pythagorean theorem. *Proceedings of International Conference on Research in Education, 1*(1), 154–162. https://doi.org/10.24071/icre.v1i1.14

Suryadi, D. (2013). Didactical design research (DDR) to improve the teaching of mathematics. *Far East Journal of Mathematical Education, 10*(1), 91–107.

Sumarsih, S. (2021). Mistakes and difficulties analysis of junior high school students on the Pythagorean theorem material and its alternative solutions. *PAEDAGOGIA*.

Sandra, Y., Suryadi, D., & Dasari, D. (2026). Epistemological, ontogenic, and didactical learning obstacles in the Pythagorean theorem: A systematic literature review. *Journal of Advanced Sciences and Mathematics Education, 6*(2).