# ANALYSIS OF STUDENTS' MATHEMATICAL REPRESENTATION ABILITY THROUGH GEOGEBRA-ASSISTED DISCOVERY LEARNING IN TERMS OF SELF-CONCEPT AND LEARNING MOTIVATION

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## Abstract

Mathematical representation ability enables students to communicate mathematical ideas through visual, symbolic, and verbal forms. This study analyzed students' mathematical representation ability through GeoGebra-assisted Discovery Learning in terms of self-concept and learning motivation. A qualitative descriptive approach involved 36 Grade XI-5 students at SMA Negeri 3 Tegal in 2025/2026, with seven students purposively selected based on self-concept and learning motivation categories. Data were collected through questionnaires, mathematical representation tests, and semi-structured interviews and analyzed using Miles, Huberman, and Saldaña's interactive model. The findings showed that mathematical representation ability varied according to self-concept and learning motivation. Students with high self-concept and high learning motivation performed well across visual, symbolic, and verbal representations, while those with lower levels experienced difficulties, particularly in constructing mathematical models and expressing ideas verbally. Symbolic representation was strongest, whereas verbal representation was most challenging. GeoGebra-assisted Discovery Learning supported representation development, while self-concept and learning motivation influenced reasoning.

Keywords: mathematical representation ability; Discovery Learning; GeoGebra; self-concept; learning motivation;

## 1 INTRODUCTION

Mathematical representation ability is the ability to express mathematical ideas through visual, symbolic, and verbal forms. NCTM (NCTM, 2000) emphasizes the use of various representations to deepen students' understanding of mathematical relationships. Ssztates that, students should be able to create, interpret, and use various representations to develop a deeper understanding of mathematical relationships. In the topic of common tangents to two circles, mathematical representation ability is essential for enabling students to accurately illustrate the relationships among circle radii, the distance between circle centers, and the position of tangent lines.

However, students' mathematical representation ability remains relatively low. A study by Diach Budiarty and Budi Utomo (2024) found that students with low mathematical ability were unable to fully meet the indicators of mathematical representation. A similar finding was reported by Sari (2022), indicating that students with low self-concept encountered difficulties in expressing mathematical ideas using visual, symbolic, and verbal representations. Furthermore, interviews with a mathematics teacher at SMA Negeri 3 Tegal indicated that Grade XI students' mathematical representation ability varied considerably and tended to be low, particularly on the topic of common tangents to two circles, which requires strong visualization skills.

One way to enhance students' mathematical representation ability is through GeoGebra-assisted Discovery Learning.One way to enhance students' mathematical representation ability is through the implementation of the Discovery Learning model supported by GeoGebra. This model encourages students to actively discover mathematical concepts through exploration and direct learning experiences (Nurzaelani et al., 2025). Discovery Learning encourages students to construct concepts independently through investigation and discovery while GeoGebra enables students to dynamically visualize mathematical objects, thereby facilitating conceptual understanding (Fatmawati, 2024).

In addition to instructional models and learning media, mathematical representation ability is also influenced by affective factors, particularly self-concept and learning motivation. Self-concept refers to an individual's perception of their own abilities (Rahman, 2012), whereas learning motivation encourages students' active engagement throughout the learning process. Studies by Sari (2022) indicate that self-concept is associated with mathematical representation ability, while Lutfi (2021) found that students with higher learning motivation tend to demonstrate better mathematical representation skills. Moreover, GeoGebra-assisted learning has been shown to enhance students' learning motivation (Fatmawati, 2024).

Previous studies have generally examined self-concept and learning motivation separately. Research by Sari (2022) focused on self-concept, whereas Lutfi (2021) investigated mathematical representation ability in relation to learning motivation. Meanwhile, Maulida and Wahyudin (2025) examined the use of digital representations in mathematics learning without considering students' affective factors. Therefore, studies integrating mathematical representation ability, self-concept, learning motivation, the Discovery Learning model, and GeoGebra in learning the topic of common tangents to two circles remain limited. Based on this gap, the present study aims to analyze students' mathematical representation ability in Discovery Learning assisted by GeoGebra in terms of self-concept and learning motivation.

## 2 METHODOLOGY

This study adopted a qualitative approach with a descriptive research design. The participants were 36 students from Class XI-5 at SMA Negeri 3 Tegal in the 2025/2026 academic year. The participants were selected through purposive sampling based on the results of the self-concept questionnaire, learning motivation questionnaire, and mathematical representation ability test. From these results, seven participants were chosen to represent different combinations of self-concept and learning motivation categories: high–high (HH), high–medium (HM), medium–high (MH), medium–medium (MM), medium–low (ML), low–medium (LM), and low–low (LL).

Data were obtained from several sources, including self-concept questionnaires, learning motivation questionnaires, mathematical representation ability tests, and semi-structured interviews. The self-concept questionnaire was constructed based on the three indicators proposed by Calhoun and Acocella, as cited in Hidayat (2020), namely knowledge, expectation, and evaluation. Meanwhile, the learning motivation questionnaire was developed according to the six indicators introduced by Hamzah B. Uno, as cited in Makhmuri (2020) and Lahagu (2025). The students' self-concept and learning motivation scores were then grouped into high, moderate, and low categories based on the categorization criteria proposed by Azwar (2013).

The mathematical representation ability test was developed by referring to three forms of representation identified by Maulyda (2020): visual, symbolic, and verbal representation. These indicators are in line with the representation standards established by the National Council of Teachers of Mathematics (2000) and have been used in previous research to identify students' mathematical representation ability (A. Maulyda & Tyaningsih, 2019). Prior to data collection, all research instruments underwent expert validation to ensure their suitability for use in the study.

The collected data were analyzed using the interactive model proposed by Miles (2014), which involves three main stages: data reduction, data display, and conclusion drawing and verification. To ensure the credibility of the findings, technical triangulation was applied by comparing information obtained from the questionnaires, mathematical representation ability tests, and semi-structured interviews (Sugiyono, 2013).

The data were analyzed using Miles' (2014) interactive model, comprising data reduction, data display, and conclusion drawing and verification. Data credibility was ensured through technical triangulation by comparing results from questionnaires, mathematical representation tests, and semi-structured interviews (Sugiyono, 2013).

# 3 RESULTS

This study aimed to examine the mathematical representation ability of Grade XI-5 students at SMA Negeri 3 Tegal by considering their self-concept and learning motivation in the context of Discovery Learning assisted by GeoGebra. Data were obtained through several research instruments, namely self-concept and learning motivation questionnaires, a mathematical representation ability test, and interviews. The findings obtained from the study are presented in the following sections.

## 3.1 Results of the Self-Concept and Learning Motivation Questionnaires

The study analyzed data obtained from the self-concept questionnaire, learning motivation questionnaire, mathematical representation ability test, and interviews conducted with the research participants. The classification of students' self-concept and learning motivation categories was determined based on the categorization criteria proposed by Azwar (2013), which classified the scores into high, moderate, and low categories. This section presents the findings on the mathematical representation ability of Grade XI-5 students at SMA Negeri 3 Tegal in terms of self-concept and learning motivation. The results of the self-concept questionnaire are presented in Table 1.

**Table 1.** *Students' Self-Concept Categories*

| Self-concept | Frequency | Persentage |
|---|---|---|
| High | 8 | 22,22% |
| Moderate | 24 | 66,67% |
| Low | 4 | 11,11% |
| Total | 36 | 100% |

As shown in Table 1, the majority of Grade XI-5 students at SMA Negeri 3 Tegal were classified as having a moderate level of self-concept. The results of the learning motivation questionnaire are presented in Table 2.

**Table 2.** *Students' Learning Motivation Categories*

| Learning Motivation | Frequency | Persentage |
|---|---|---|
| High | 7 | 19,44% |
| Moderate | 23 | 63,89% |
| Low | 6 | 16,67% |
| Total | 36 | 100% |

As shown in Table 2, the majority of students fell into the moderate category for learning motivation. Following the completion of the self-concept and learning motivation questionnaires, the students also took a mathematical representation ability test. The test findings showed differences in students' levels of mathematical representation ability. Of the 36 students who participated in the study, the highest score obtained was 97.72, while the lowest was 40.90. A score of 75 occurred most frequently among the participants.

The questionnaire results were then used to classify students according to their self-concept and learning motivation levels, which subsequently served as a basis for determining the research participants. The mathematical representation ability test results were also considered to provide a more detailed description of the selected participants' abilities. The participants selected for further analysis are presented in Table 4.

**Table 3.** *Selected Research Participants*

| No. | Participant | Self-concept | Learning Motivation |
|---|---|---|---|
| 1 | HH | High | High |
| 2 | HM | High | Moderate |
| 3 | MH | Moderate | High |
| 4 | MM | Moderate | Moderate |
| 5 | ML | Moderate | Low |
| 6 | LM | Low | Moderate |
| 7 | LL | Low | Low |

The mathematical representation ability test revealed noticeable differences in the students' performance. Of the 36 students who took the test, scores ranged from 40.90 to 97.72, with 75 being the score obtained most frequently. Students' mathematical representation ability was examined using three indicators, namely visual, symbolic, and verbal representation, adapted from Maulyda and Tyaningsih (2019).

**Table 5.** *Indicators of Mathematical Representation Ability*

| No. | Representation | Operational sentence |
|---|---|---|
| 1 | Visual | (R1) Presenting data or information in an appropriate visual representation (graphical representation) that reflects the student's internal representation. |
| 2 | Symbolic | (R2) Constructing mathematical equations or models from the given representation in accordance with the student's internal representation. |

| No. | Representation | Operational sentence |
|---|---|---|
| 3 | Verbal | (R3) Expressing the interpretation of a mathematical problem representation in appropriate words that reflect the student's internal representation. |

Each participant, representing different levels of self-concept and learning motivation, demonstrated distinct characteristics of mathematical representation ability. These characteristics were identified through the analysis of students' written test responses and in-depth interviews in accordance with the three indicators of mathematical representation ability

## 3.2 Results of the Mathematical Representation Ability Test

The results of the analysis of students' mathematical representation ability are presented in Table 6.

**Tabel 6.** *Result Of Mathematical Representation Ability*

| No. | Participant | Visual | Symbolic | Verbal |
|---|---|---|---|---|
| 1. | HH | ✓ | ✓ | ✓ |
| 2. | HM | - | ✓ | ✓ |
| 3. | MH | - | - | - |
| 4. | MM | ✓ | ✓ | ✓ |
| 5. | ML | * | * | * |
| 6. | LM | ✓ | * | ✓ |
| 7. | LL | * | - | - |

Notes :

✓ : “Able”

* : “Fairly Able”

- : “Less Able”

Students’ mathematical representation ability was examined using three indicators identified by Maulyda and Tyaningsih (2019), namely visual, symbolic, and verbal representation. The results showed that students with varying levels of self-concept and learning motivation exhibited different characteristics in their mathematical representation ability when working with the topic of common tangent lines of two circles across these three forms of representation.

### *3.2.1* Mathematical Representation Ability of Students with High Self-Concept and High Learning Motivation

In terms of visual representation, students accurately illustrated essential elements, such as circles, center points, and tangent lines. Interview results indicated that GeoGebra supported the visualization of geometric relationships, making it easier for students to construct visual representations. For symbolic representation, students successfully translated problems into mathematical models, selected appropriate formulas, and obtained correct solutions. These findings are in line with earlier research, which has shown that positive self-concept and high learning motivation support stronger visual and symbolic representation abilities (Sari & Manoy, 2022; Lutfi & Khusna, 2021; Maulida & Wahyudin, 2025). Regarding verbal representation, students were able to explain their solutions, although some inaccuracies in mathematical terminology were still observed. Overall, students with high self-concept and high learning motivation demonstrated good mathematical representation ability, particularly in visual and symbolic representation. However, further practice is needed to improve the accuracy of verbal mathematical communication.

### *3.2.2* Mathematical Representation Ability of Students with High Self-Concept and Moderate Learning Motivation

Based on the written test and interview results, students with moderate self-concept and high learning motivation (MH) demonstrated fairly good mathematical representation ability. In visual representation (R1), students were able to present information graphically but occasionally omitted essential geometric elements. The use of GeoGebra during Discovery Learning helped students visualize the relationships among circles and tangent lines, supporting their understanding of geometric concepts. For symbolic representation (R2), students were able to construct mathematical models and apply appropriate formulas systematically. Although their moderate self-concept sometimes reduced their confidence when solving complex problems, their high learning motivation encouraged them to persist throughout the Discovery Learning process. However, verbal representation (R3) remained the weakest indicator, as students experienced difficulties in expressing mathematical interpretations accurately in written form. Overall, students with moderate self-concept and high learning motivation demonstrated stronger symbolic representation than visual and verbal representation, indicating that further practice is needed to improve their written mathematical communication.

For symbolic representation (R2), students successfully constructed mathematical models, selected appropriate formulas, and completed calculations systematically. The stages of Discovery Learning encouraged students to identify problem information, formulate mathematical models, and verify their solutions, making symbolic representation their strongest indicator. In contrast, verbal representation (R3) remained less developed, as students experienced difficulties in expressing mathematical ideas in written form despite being able to explain the concepts orally during interviews. Overall, students with high self-concept and moderate learning motivation showed strengths in symbolic representation, while their visual and verbal representations still require improvement. These findings are consistent with previous studies (Sari & Manoy, 2022; Lutfi & Khusna, 2021; Maulida & Wahyudin, 2025)

#### *3.2.3* Mathematical Representation Ability of Students with Moderate Self-Concept and High Learning Motivation

The written test and interview findings showed that students with moderate self-concept and high learning motivation (MH) had a relatively good level of mathematical representation ability.In visual representation (R1), students were able to present information graphically but occasionally omitted essential geometric elements. The use of GeoGebra during Discovery Learning helped students visualize the relationships among circles and tangent lines, supporting their understanding of geometric concepts. For symbolic representation (R2), students were able to construct mathematical models and apply appropriate formulas systematically. Although their moderate self-concept sometimes reduced their confidence when solving complex problems, their high learning motivation encouraged them to persist throughout the Discovery Learning process. However, verbal representation (R3) remained the weakest indicator, as students experienced difficulties in expressing mathematical interpretations accurately in written form. Overall, students with moderate self-concept and high learning motivation demonstrated stronger symbolic representation than visual and verbal representation, indicating that further practice is needed to improve their written mathematical communication.

#### *3.2.4* Mathematical Representation Ability of Students with Moderate Self-Concept and Moderate Learning Motivation

The implementation of Discovery Learning assisted by GeoGebra also provided opportunities for students to explore geometric relationships and construct mathematical representations on the topic of common tangent lines of two circles. These results are in agreement with earlier research, which suggests that incorporating GeoGebra into Discovery Learning can help strengthen students' conceptual understanding and mathematical performance.

Based on the written test and interview results, students with moderate self-concept and moderate learning motivation (MM) demonstrated good mathematical representation ability across the three indicators. In R1, students were able to present information in appropriate graphical representations, although some visual details were still incomplete. The implementation of Discovery Learning assisted by GeoGebra on the topic of common tangent lines of two circles supported students in visualizing geometric relationships, making it easier to construct representations. In R2, students successfully constructed mathematical models from the given information and completed the solution systematically, indicating that symbolic representation was their strongest indicator. However, in R3, students were able to interpret their solutions in words, although their explanations were sometimes incomplete. The findings also support earlier research indicating that self-concept and learning motivation contribute to the development of students' mathematical representation ability. (Sari & Manoy, 2022; Lutfi & Khusna, 2021; Maulida & Wahyudin, 2025)

#### *3.2.5* Mathematical Representation Ability of Students with Moderate Self-Concept and Low Learning Motivation

Students categorized as having moderate self-concept and low learning motivation (ML) showed a moderate level of mathematical representation ability. In R1, they were able to present basic graphical representations but still omitted several essential geometric elements. In R2, students attempted to construct mathematical models; however, errors in selecting formulas and performing calculations were still evident. Meanwhile, R3 was the weakest indicator, as students experienced difficulties in interpreting and communicating mathematical ideas in written form. Although Discovery Learning assisted by GeoGebra supported students in exploring geometric concepts and constructing representations, their low learning motivation limited their engagement throughout the learning process. The results also support earlier research suggesting that students with lower levels of learning motivation tend to demonstrate weaker mathematical representation ability (Sari & Manoy, 2022; Lutfi & Khusna, 2021).

#### *3.2.6* Mathematical Representation Ability of Students with Low Self-Concept and Moderate Learning Motivation

Students in the low self-concept and moderate learning motivation (LM) category showed a moderate level of mathematical representation ability. In R1, they were generally able to present appropriate graphical representations, while in R2, they successfully constructed mathematical models despite making occasional computational errors. However, R3 remained the weakest indicator, as students experienced difficulties in expressing mathematical interpretations systematically in written form, which is consistent with the findings of Lutfi and Khusna (2021). Although Discovery Learning assisted by GeoGebra supported students in exploring geometric relationships on the topic of common tangent lines of two circles, their low self-concept limited their confidence in communicating mathematical ideas. This finding is in line with Sari and Manoy (2022) reported that students who hold a more positive view of themselves tend to exhibit stronger mathematical representation ability.

#### *3.2.7* Mathematical Representation Ability of Students with Low Self-Concept and Low Learning Motivation

Students categorized as having both low self-concept and low learning motivation (LL) showed the weakest mathematical representation ability across all three indicators. In R1, they were unable to present complete graphical representations that reflected the given information. In R2, they attempted to construct mathematical models but experienced difficulties in selecting appropriate formulas and completing the solution accurately. R3 was the weakest indicator, as students were unable to interpret or communicate their mathematical reasoning clearly in written form, which is consistent with the findings of Lutfi and Khusna (2021). Similarly, Sari and Manoy (2022) reported that students with less positive self-concept generally showed lower levels of mathematical representation ability.

## 4 CONCLUSIONS

Based on the findings, conclusions regarding students' mathematical representation ability through GeoGebra-assisted Discovery Learning in terms of self-concept and learning motivation are presented below.

1. They successfully demonstrated the visual, symbolic, and verbal representation indicators. Nevertheless, their verbal responses still contained a few minor inaccuracies in the use of mathematical terminology.
2. Students with high self-concept and moderate learning motivation (HM) demonstrated fairly good mathematical representation ability. Symbolic representation was their strongest indicator, while visual representation still required greater accuracy, and verbal representation had not yet developed optimally.
3. Students with moderate self-concept and high learning motivation (MH) demonstrated fairly good mathematical representation ability. They were able to construct visual and symbolic representations; however, they still experienced difficulties in interpreting solutions in written form, resulting in less optimal verbal representation.
4. Students with moderate self-concept and moderate learning motivation (MM) demonstrated good mathematical representation ability. They were able to present visual and symbolic representations with reasonable accuracy, although they still had limitations in communicating mathematical ideas in writing, particularly in the verbal representation indicator.
5. Students with moderate self-concept and low learning motivation (ML) demonstrated moderately developed mathematical representation ability. They were able to present visual and symbolic representations procedurally but still encountered difficulties in conceptual accuracy and verbal interpretation.

6. Students with low self-concept and moderate learning motivation (LM) demonstrated moderately developed mathematical representation ability. They were able to construct visual representations and mathematical models for most problems; however, verbal representation remained their weakest indicator.
7. Students classified as having both low self-concept and low learning motivation (LL) exhibited the lowest level of mathematical representation ability. They encountered difficulties in fulfilling the three representation indicators, especially when producing visual representations, developing mathematical models, and verbally explaining or interpreting their solutions.

Overall, the findings indicate that students' mathematical representation ability in Discovery Learning assisted by GeoGebra varied across different combinations of self-concept and learning motivation. In general, students with higher levels of self-concept and learning motivation tended to perform better in visual, symbolic, and verbal representation. Conversely, students with lower self-concept and learning motivation showed greater difficulties in constructing visual representations, developing mathematical models, and expressing mathematical ideas verbally. These findings indicate that self-concept and learning motivation are associated with differences in students' mathematical representation ability during Discovery Learning assisted by GeoGebra.

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